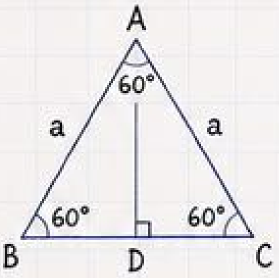
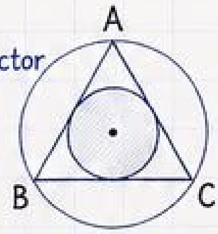
✧ Geometry – Special Triangles & Applications ✧

Equilateral Triangle



- All sides equal
- All angles = 60°
- All centres I, O, G, H coincide at same point.
  AD is median, altitude, angle bisector, perpendicular bisector



Height: $h = \frac{\sqrt{3}}{2}a$ Area: $\text{Area} = \frac{\sqrt{3}}{4}a^2$

Circumradius: $R = \frac{a}{\sqrt{3}} = \frac{2}{3}h$ Inradius: $r = \frac{a}{2\sqrt{3}} = \frac{h}{3}$

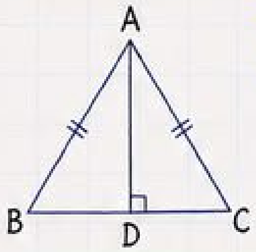
$\boxed{R : r = 2:1}$ $\boxed{\text{Area (circumcircle)} : \text{Area (incircle)} = 4:1}$

Equilateral Triangle Relations

Side	:	Height	:	Area
a	:	$\frac{\sqrt{3}}{2}a$	:	$\frac{\sqrt{3}}{4}a^2$

If side = $2k \rightarrow$ height = $\sqrt{3}\,k \rightarrow$ area = $\sqrt{3}\,k^2$

Isosceles Triangle

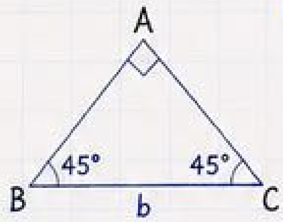


- Two sides equal (AB = AC)
- All four centres lie on AD

Height: $AD = \sqrt{a^2 - \frac{b^2}{4}}$

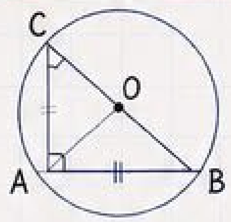
Area = $\frac{b}{4}\sqrt{4a^2 - b^2}$

Isosceles Right Triangle



- Angles: 45°, 45°, 90°
  If hypotenuse = H
  Legs = $\frac{H}{\sqrt{2}}$
  Area = $\frac{H^2}{4}$
  Perimeter = $H(\sqrt{2}+1)$

Right Angle Triangle



- One angle = 90°
- Inscribed in a semicircle

Relations: $r = \frac{P+B-H}{r+R} = \frac{P+B}{2}$

$R = BO = \text{shortest median} = \frac{H}{2}$

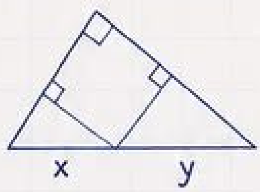
$BG = \frac{H}{3}$, $GO = \frac{H}{6}$

Area: $\text{Area} = r \times S = S(S-2R) = r^2 + 2rR$

Pythagorean Triplets

Odd number method → (3,4,5), (5,12,13)
Even number method → (6,8,10), (8,15,17)

Square Inscribed in a Triangle



If base segments = x and y

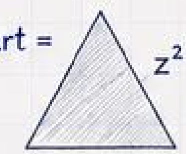
Side of square: $a = \frac{xy}{x+y}$

In right triangle: Side = $\frac{ab}{a+b}$

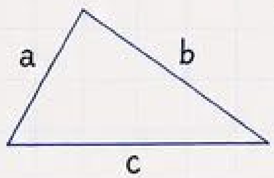
Side (largest square): $y = \frac{abc}{a^2+b^2+ab}$, where $x > y$

Area Results

If $x^2 + y^2 = z^2$
Area of shaded part =
Area of ΔABC



Scalene Triangle



- All sides unequal
  Perimeter: $P = a + b + c$
  Semi-perimeter: $s = \frac{a+b+c}{2}$

Area: $\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

$\text{Area} = r \times s$

$\text{Area} = \frac{abc}{4R}$